Honley CE (VC) Junior School

WRITTEN CALCULATIONS POLICY (updated to reflect National Curriculum 2014)

Edited by Isobel Gabanski	June 2016
Reviewed and approved by governors	Jan 2014
Next Review Date	June 2017

Updated 17.6.16

INTRODUCTION

The methods shown here are the way we do it at OUR school.

- we believe these methods teach deep understanding, so children don't get stuck later.
- all our work uses visual representations (eg, numberlines) and equipment (eg, multibase) so that children can REMEMBER the visual and physical things they have seen/done.
- many methods will be familiar, but the WAY we teach them and the MATHS LANGUAGE we use is extremely important.
- We ALWAYS talk of the digits in terms of their value. 235: that isn't 3, it's 3 tens, or 30.
- by the time we introduce these methods, we are assuming a certain level of competence already; competence in a variety of mental methods, a good understanding of place value and number facts, and of how the four operations relate to one another.

We always follow:

concrete (using objects) → **pictorial** (diagrams) → **abstract** (numbers and symbols only)

Addition

```
  23454
+   596
  24050
   111
```

Subtraction

```
     1
    2 3 1
  5 2 3 4 4
-   1 1 8 7
  5 1 1 5 7
```

Long multiplication

```
  5172
x   38
155160
 41376
196536
   1
```

Short division

```
     4 3 r5
13 | 5 6 ⁴4
```

$564 \div 13 = 43 \text{ r } 5 = 43 \frac{5}{13}$

Chunking

```
         28
15 | 432
   - 300      (20 x 15)
     132
   - 120      (8 x 15)
      12
```

These are the methods we are working towards.

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ADDITION AND SUBTRACTION

When Are Children Ready For Written Calculations?

- Do they know addition and subtraction facts to 20?
- Do they understand place value and can they partition numbers?
- Can they add three single digit numbers mentally?
- Can they add and subtract any pair of two digit numbers mentally?
- Can they explain their mental strategies orally and record them using informal jottings?
- Do they use and apply the commutative (any order) and associative (grouping some numbers together makes it easier to calculate sometimes) laws of addition?

The above list is not exhaustive but is a guide for the teacher to judge when a child is ready to move from informal to formal methods of calculation.

<table>
<tr><th colspan="4">What is the expected size of numbers each year group should be working with in <u>written</u> calculations? (as directed by the National Curriculum 2014)</th></tr>
<tr><th colspan="2">Y1</th><th colspan="2">Y2</th></tr>
<tr><td colspan="2">Read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs</td><td colspan="2">Add and subtract two two-digit numbers using concrete objects, pictorial representations progressing to formal written methods

46
+ 27
73
1

6 1
73
- 46
27</td></tr>
<tr><th>Y3</th><th>Y4</th><th>Y5</th><th>Y6</th></tr>
<tr><td>Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

423
+ 88
511
1 1

2 3 1
344
- 187
157</td><td>Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition where appropriate

2458
+ 596
3054
1 1 1

1
2 3 1
2344
- 187
2157</td><td>Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)

23454
+ 596
24050
1 1 1

1
2 3 1
52344
- 1187
51157</td><td>Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
At this stage it is not the size of the number, but more the complexity of the problem that is relevant (eg, multi-step problems)</td></tr>
</table>

NB: this policy is written with <u>most</u> children in mind. Some methods may need to be adapted to suit individuals.

Written Addition

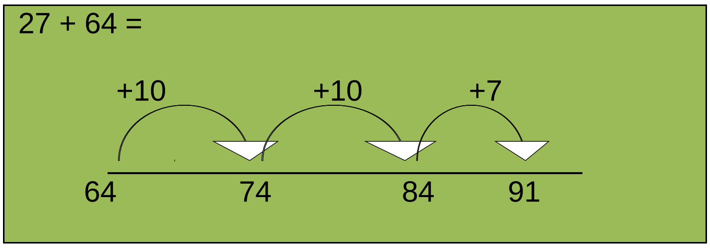
In KS1, children are taught to partition <u>both</u> numbers; some children will move on to partitioning just one. We encourage partitioning of one number only.

The methods we use are:

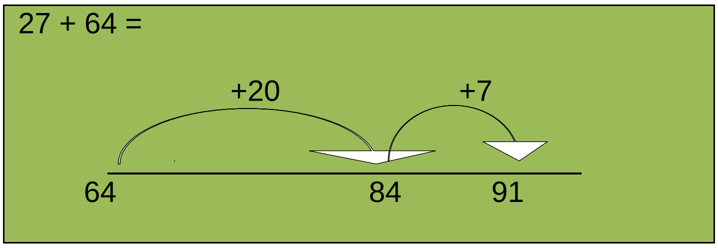
- number lines
- pictorial and abstract together
- abstract only

<u>Number lines</u>

Counting on from the larger number in tens then ones, eg, 64 + 27

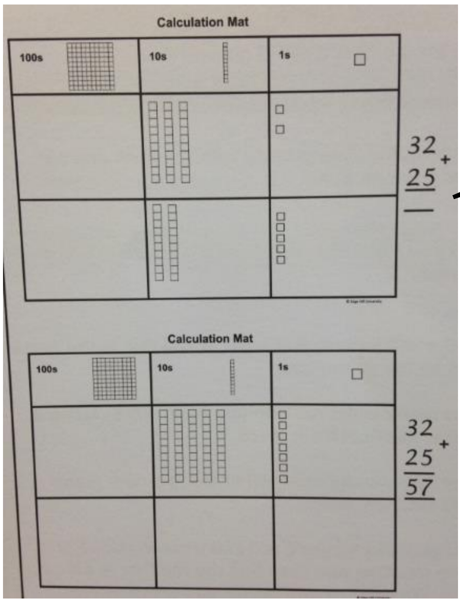


Develop this from moving in individual steps of 10 to more efficiently adding the tens in one jump.



Use this method with larger numbers too.

<u>Pictorial and abstract together</u>



Use a calculation mat such as this with multibase, and write the corresponding sum vertically alongside.

Start with the <u>**least significiant digit**</u> first.

Note the terminology: “ones” is sometimes used instead of “units”.

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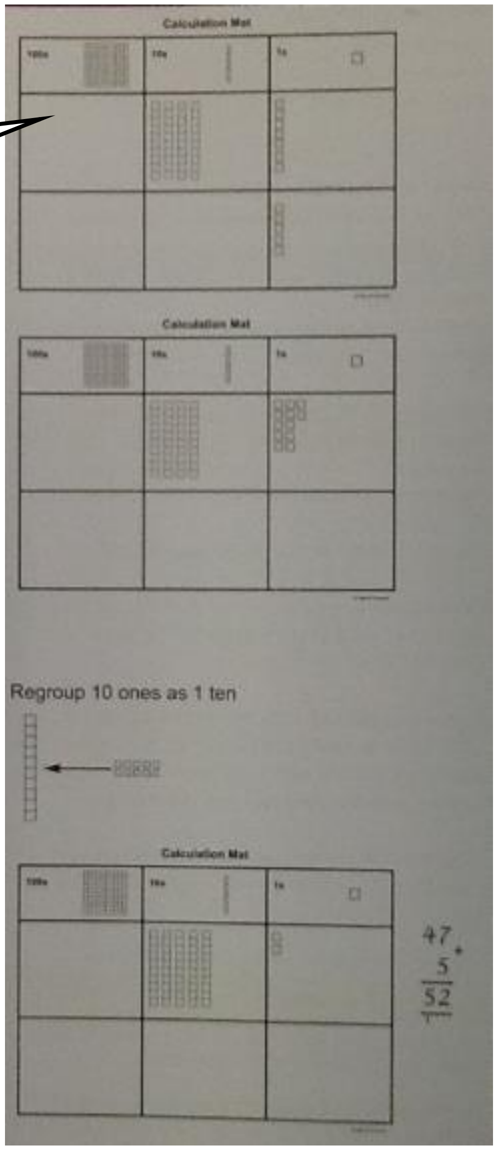
The next stage: **2 digit numbers with some regrouping**. Again, keep the written sum alongside.

DO NOT refer to borrowing or carrying - use the term "regrouping".

The next stage: **2 digit numbers with further regrouping**, eg, 77 + 45.

Apply to larger numbers and in the context of money, measures etc.



<u>Abstract only</u>

The next stage is purely abstract (numbers only, no mats). This method is used with all sizes of numbers - see the National Curriculum expectations for the end of each year group.

```
   423
+   88
   511
   1 1
```

```
   23454
+    596
   24050
    111
```

Written Subtraction

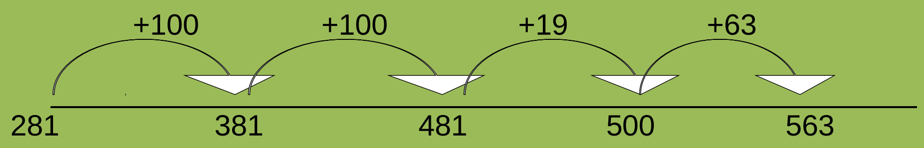
In KS1, the children have counted up to the next whole 10 first of all; we will count in 10s or 100s first of all.

The methods we use are:

- number lines
- pictorial and abstract together
- abstract only

Number lines - counting up:

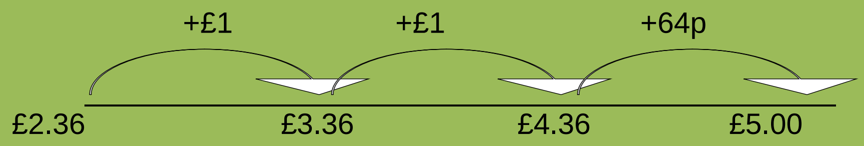
563 - 281 =



Answer 200 + 63 +19 = 282

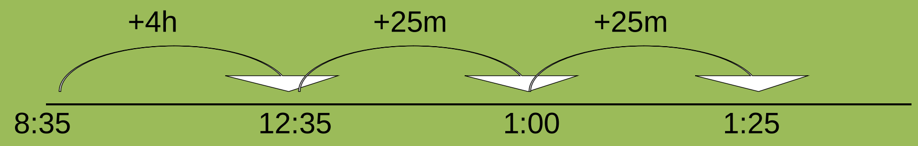
Calculating with numbers with more than 4 digits, money, time, decimals - use the same principle.

Money   £5 - £2.36 =



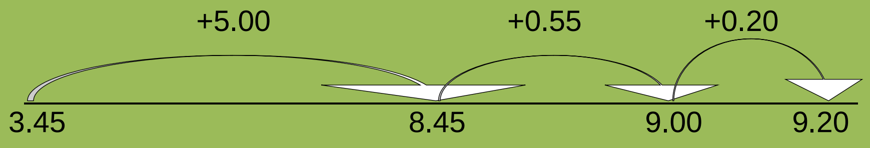
Answer £2.00 + 64p = £2.64

Time   8:35 to 1:25



Answer 4h 50m

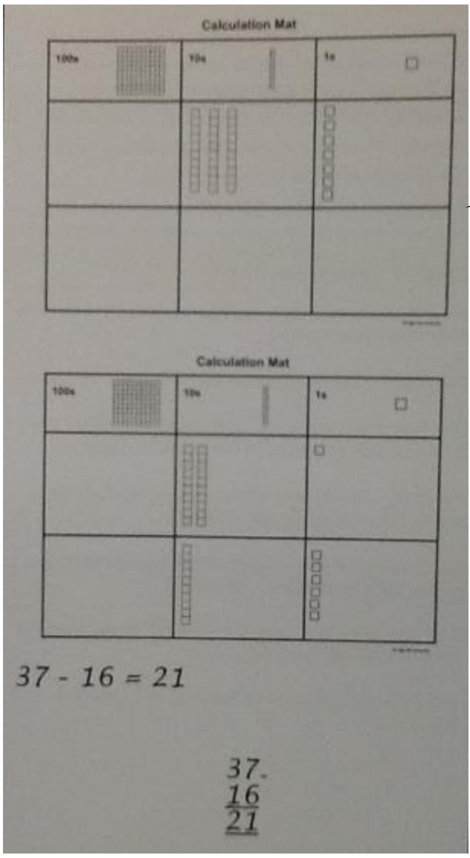
Decimals   9.2 - 3.45 =



Answer 5.00 + 0.55 + 0.20 = 5.75

Pictorial and abstract together, including some regrouping

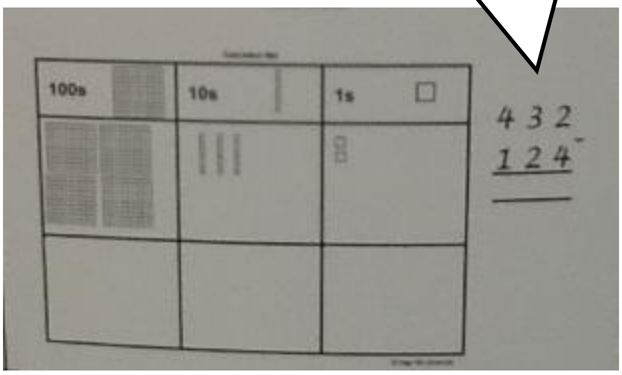
We are starting with the least significant digit.



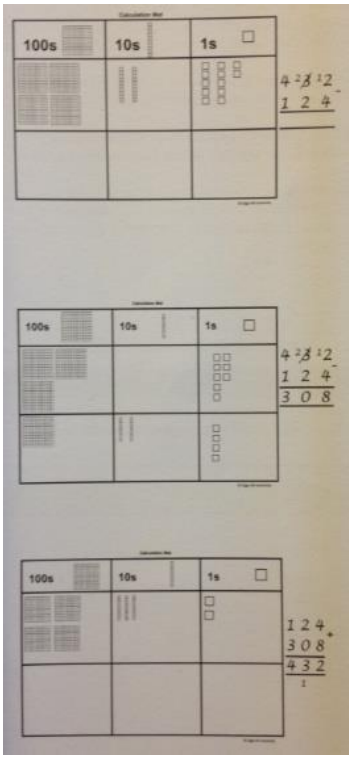
Use calculation mats and multibase, writing the sum in numerals alongside.

No regrouping is needed in this one.

This one needs regrouping.



We don't call it borrowing anymore! On the calculation mat, with the equipment, the tens rods would have to be REGROUPED into single cubes, hence the term.



Model the regrouping with multibase alongside the written form of the sum.

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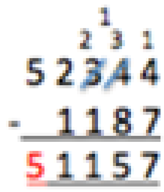
Abstract only

The next stage is purely abstract (numbers only, no mats). This method is used with all sizes of numbers - see the National Curriculum expectations for the end of each year group.



MULTIPLICATION AND DIVISION

When are children ready for written multiplication and division calculations?

- Do they know the 2, 3, 4, 5 and 10 times table
- Do they know the result of multiplying by 0 and 1?
- Do they understand 0 as a placeholder?
- Can they multiply two and three digit numbers by 10 and 100?
- Can they double and halve two digit numbers mentally?
- Can they use multiplication facts they know to derive mentally other multiplication facts that they do not know?
- Can they explain their mental strategies orally and record them using informal jottings?
- Do they use the commutative (any order) and associative (grouping some numbers together makes it easier to calculate sometimes) laws for multiplication and the distributive (breaking up or combining numbers to make a calculation easier) law of multiplication over addition and subtraction?
- Do they recognise that multiplication and division are inverse operations?

The above list is not exhaustive but is a guide for the teacher to judge when a child is ready to move from informal to formal methods of calculation.

Written Multiplication

What is the expected size of numbers each year group should be working with in written calculations? (as directed by the National Curriculum 2014)			
Y1	**Y2**		
	Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs		
Y3	**Y4**	**Y5**	**Y6**
Write and calculate mathematical statements for ÷ using the x tables they know progressing to formal written methods.	Multiply two-digit and three-digit numbers by a one-digit number using formal written layout 243 x 6 2058 1	Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers 243 x 36 7290 1458 8748 1	Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication 5172 x 38 155160 41376 196536 1

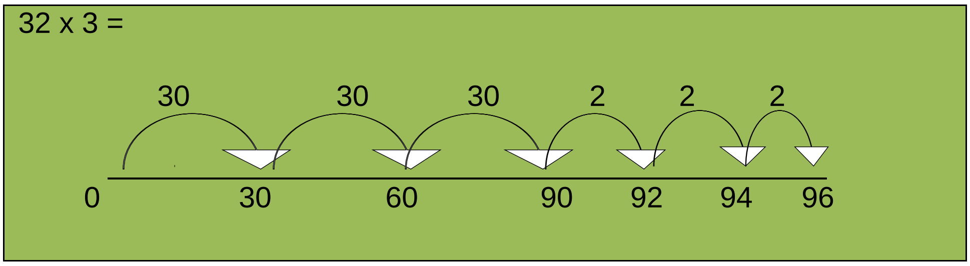
NB: this policy is written with most children in mind. Some methods may need to be adapted to suit individuals.

In KS1, the children have used lots of arrays to show commutativity (they count rows first and then columns). They do pictorial groupings first, then jumps along a number line. Following that they have then introduced the x sign with numerical representation, then arrays and sums alongside (eg, 4x3).

The methods we use are:

- Number lines and horizontal partitioning
- Brackets
- Grid method with the formal algorithm alongside
- Just the formal algorithm

<u>Number lines and horizontal partitioning:</u>



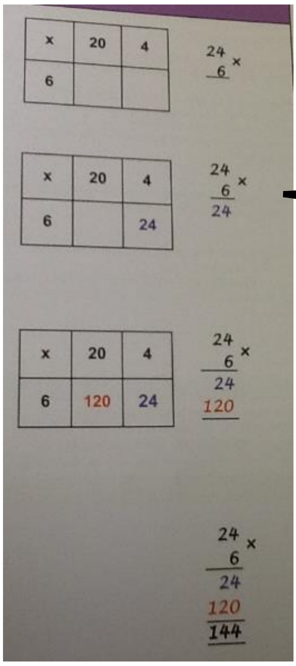
<u>Using brackets:</u>

32 x 3 = (30 x3) + (2 x 3) = 90 + 6 = 96

<u>Grid layout:</u>

Remember: if children are using this method they should already be able to multiply a 1-digit number by a multiple of 10 (eg, 3 x 70).

Show the algorithm alongside the grid method.



2-digit x 1-digit
Start with multiplying the ones digit first.

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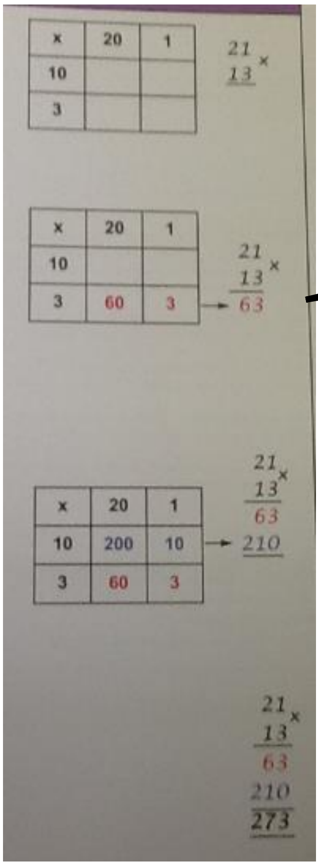
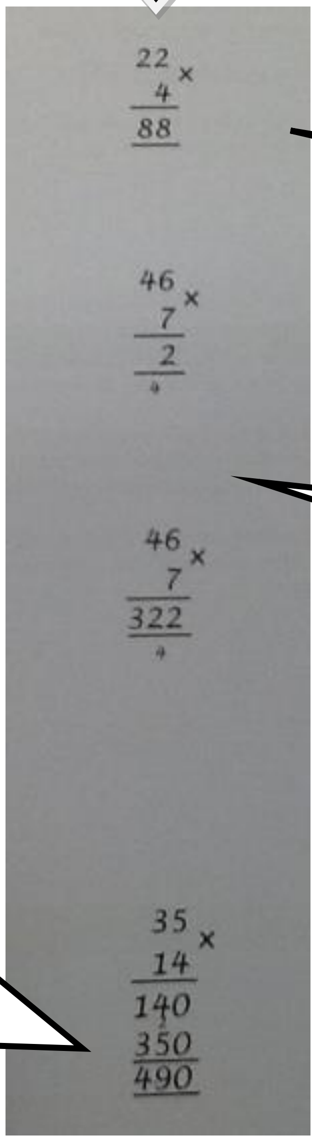


2-digit x 2-digit
Start with multiplying the ones digit first.

Extend the method to sums that do not have the pictorial support of the grid method alongside.



No regrouping.

Regrouping of ones into tens.

Formal algorithm

2-digit x 2-digit
Start with the ones: 4x 5 are 20; put the 0 in the ones column and the regrouped 2 has to go in the tens.
Then 4 x 30 is 120; add on the extra 2 regrouped tens (120 + 20), so we get 140.

Then 10x35 = 350 (by this stage, children should be able to multiply any number by 10).

Then add together.

Extend the principle to larger numbers and decimals.

Written Division

What is the expected size of numbers each year group should be working with in <u>written</u> calculations? (as directed by the National Curriculum 2014)			
Y1	**Y2**		
	Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs		
Y3	**Y4**	**Y5**	**Y6**
Write and calculate mathematical statements for ÷ using the x tables they know progressing to formal written methods. Write and calculate mathematical statements for ÷ using the x tables they know progressing to formal written methods.	*"Chunking up" on a number line* *196 ÷ 6 = 32 r 4*	Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context 3 2 $6\overline{)1\;9\;^{1}2}$	Divide numbers up to 4-digits by a two-digit whole number using the formal written method of short division, or long division, and interpret remainders as whole number remainders, fractions, or by rounding where appropriate for the context 4 3 r 5 $13\overline{)5\;6\;^{4}4}$ $564 \div 13 = 43 \text{ r } 5 = 43\frac{5}{13}$ 28 15 | 432 - <u>300</u> (20 x 15) 132 - <u>120</u> (8 x 15) 12

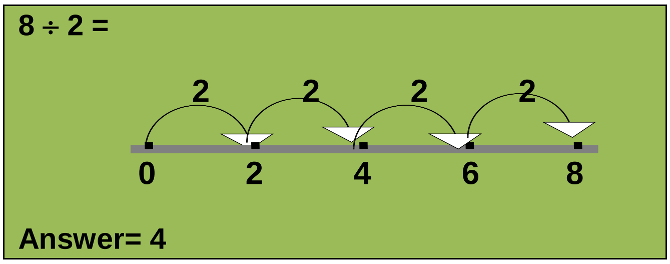
KS1 start with sharing, eg 8 sweets shared between 2 friends, how many would there be each? Then they move on to grouping: eg, 8 sweets in 2 groups, how many groups could we have? Following this, they use a labelled number line on which to count/up or down, developing further to using a blank number line, then including remainders.

The methods we use are:

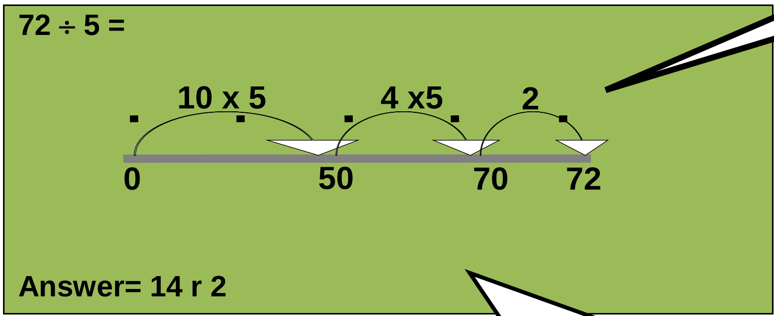
- Numberlines, including chunking
- Long division with chunking
- Short division

Number lines and the beginnings of chunking

Lots of number line work, extending to larger numbers.



This is **“chunking”.**



An earlier step to this is showing **all** the jumps of 5; here we have done 10 jumps in one go, and then another 4 in one go. Children ought to begin to suggest this sort of shortcut as a more efficient way of solving the problem.

Play around with different step sizes, eg would it still work if we did 5 x 5 and then 2 x 5 etc....? Does the size of the step matter?

An advanced understanding is when children begin to use steps that are multiples of 10. Eg, in the sum **786 ÷ 25,** some children may begin to recognise they can do 30 x 25 as their first step.

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Long division using Chunking

72 ÷ 5 =

```
        14 r 2
   5 | 72
      -50         (10 x 5)
       22
       20          (4 x 5)
        2
```

Answer: 14 r 2

2-digit number ÷ single digit with remainder

256 ÷ 7 =

```
         36 r 4
   7 | 256
      -210         (30 x 7)
        46
       -42          (6 x 7)
         4
```

Answer: 36 r 4

3-digit number ÷ single digit with remainder

87.5 ÷ 7 =

```
        12.5
   7 | 87.5
      -70.0         (10 x 7)
       17.5
       14.0          (2 x 7)
        3.5
      - 3.5   (0.5 x 7)
```

Answer: 12.5

2-digit number ÷ single digit with **decimal** remainder

This involves transference of knowledge between multiplication tables and the decimal counterpart which is 10x smaller. Eg, in the 7x table, 5 x 7 = 35, so 0.5 x 7 = 3.5.

This is just the same as knowing that if 5 x 7 = 35, then 50 x 70 = 350 (being 10x bigger).

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432 ÷ 15 =

```
          28
15  | 432
    - 300        (20 x 15)
      132
    - 120        (8 x 15)
       12
```

3-digit number ÷ 2-digit with **fraction** remainder

If we need the answer as a fraction, we take the remainder and the divisor: $\frac{12}{15} = \frac{4}{5}$

So the final answer to the sum is: $28\frac{4}{5}$

Sometimes the answer may need to be expressed as a decimal, so this would be 28.8. The child would need to know fraction/decimal equivalents or how to work them out.

Short division

98 ÷ 7 becomes	432 ÷ 5 becomes	496 ÷ 11 becomes
1 4	8 6 r 2	4 5 r 1
7 | 9 ^{2}8	5 | 4 3 ^{3}2	1 1 | 4 9 ^{5}6
Answer: 14	Answer: 86 remainder 2	Answer: $45\frac{1}{11}$

In this example we would say "how many sevens are in 9?" Therefore we are not referring to the place value of the digits. This makes it even more important to make approximations/estimates of answers, and to check if an answer is realistic.

Mathematics Appendix 1: Examples of formal written methods for addition, subtraction, multiplication and division

This appendix sets out some examples of formal written methods for all four operations to illustrate the range of methods that could be taught. It is not intended to be an exhaustive list, nor is it intended to show progression in formal written methods. For example, the exact position of intermediate calculations (superscript and subscript digits) will vary depending on the method and format used.

For multiplication, some pupils may include an addition symbol when adding partial products. For division, some pupils may include a subtraction symbol when subtracting multiples of the divisor.

Addition and subtraction

789 + 642 becomes

		7	8	9
+		6	4	2
	1	4	3	1
		1	1	

Answer: 1431

874 − 523 becomes

	8	7	4
−	5	2	3
	3	5	1

Answer: 351

932 − 457 becomes

	8	1 2	1
	~~9~~	~~3~~	2
−	4	5	7
	4	7	5

Answer: 475

932 − 457 becomes

		1	1
	9	3	2
−	~~4~~	~~5~~	7
	5	6	
	4	7	5

Answer: 475

Short multiplication

24 × 6 becomes

		2	4
×			6
	1	4	4
		2	

Answer: 144

342 × 7 becomes

		3	4	2
×				7
	2	3	9	4
		2	1	

Answer: 2394

2741 × 6 becomes

		2	7	4	1
×					6
	1	6	4	4	6
		4	2		

Answer: 16 446

Long multiplication

24 × 16 becomes

		2	
		2	4
	×	1	6
	2	4	0
	1	4	4
	3	8	4

Answer: 384

124 × 26 becomes

		1	2	
		1	2	4
×			2	6
	2	4	8	0
		7	4	4
	3	2	2	4
	1	1		

Answer: 3224

124 × 26 becomes

		1	2	
		1	2	4
×			2	6
		7	4	4
	2	4	8	0
	3	2	2	4
	1	1		

Answer: 3224

Short division

98 ÷ 7 becomes

	1	4
7	9	^{2}8

Answer: 14

432 ÷ 5 becomes

		8	6	r 2
5	4	3	^{3}2	

Answer: 86 remainder 2

496 ÷ 11 becomes

			4	5	r 1
1	1	4	9	^{5}6	

Answer: $45\frac{1}{11}$

Long division

432 ÷ 15 becomes

			2	8	r 12
1	5	4	3	2	
		3	0	0	
		1	3	2	
		1	2	0	
			1	2	

Answer: 28 remainder 12

432 ÷ 15 becomes

			2	8	
1	5	4	3	2	
		3	0	0	15×20
		1	3	2	
		1	2	0	15×8
			1	2	

$\frac{12}{15} = \frac{4}{5}$

Answer: $28\frac{4}{5}$

432 ÷ 15 becomes

			2	8 ·	8
1	5	4	3	2 ·	0
		3	0	↓	
		1	3	2	↓
		1	2	0	↓
			1	2	0
			1	2	0
					0

Answer: 28·8

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